In search of relativistic time

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Abstract

This paper explores the status of some notions which are usually associated to time, like datations, chronology, durations, causality, cosmic time and time functions in the Einsteinian relativistic theories. It shows how, even if some of these notions do exist in the theory or for some particular solution of it, they appear usually in mutual conflict: they cannot be synthesized coherently, and this is interpreted as the impossibility to construct a common entity which could be called *time*. This contrasts with the case in Newtonian physics where such a synthesis precisely constitutes Newtonian time.
After an illustration by comparing the status of time in Einsteinian physics with that of the vertical direction in Newtonian physics, I will conclude that there is no pertinent notion of time in Einsteinian theories.

Keywords

time, causal structure, proper duration, cosmic time, spacetime

1 The vanishing of time

The goal of this paper is to develop and to justify the affirmation that Einsteinian relativity is the theory of the vanishing of time; more exactly, of the replacement of space and time by space-time. Quoting Hermann Minkowski (1864 - 1909), " The views of space and time which I wish to lay down for you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. " (80th Assembly of the German Natural Scientists and Physicians, Köln, september 21, 1908). This is true already for the *special* theory of relativity, and this is its central property. This is still more crucial for the *general* theory of relativity.

The analysis presented here offers no original idea (excepted the analogy in section 3.1). Everything can be found in manuals or in some epistemological analyses concerning the Einsteinian relativistic theories.[1] However, this literature is in general not concerned about the status of the notions analyzed here, and about the status of time in Einsteinian physics (see however the papers of Rovelli and Anderson in the references, whose conceptions agree with those presented here although with a different focus, more concerned with the attempts to quantize gravity; and the work of Barbour [3], who defends an original position which goes however beyond the frame of Einsteinian physics). The goal here is to offer a synthesis and a panorama of the notions which could be claimed to be linked to time, and to give a clear explanation of what is meant by the disappearance of time in Einsteinian theories.

This clarification appears desirable since one finds often in the literature references to *time* in the analysis of a relativistic situation. This short way to refer to a time function or a proper duration may give the false impression that such notions have the properties that we usually attribute to time. Apart from being a possible source of false statements, this may orient an ontological analysis into erroneous directions. Thus I estimate that this review may be especially useful for philosophers.

The paper analyzes the notions of datation, chronology, durations, causality and cosmic time, both in the Newtonian and Einsteinian theories. In Newtonian physics, these notions can be synthesized to form " Newtonian time ". Here, I defend the conclusion that, even if some of these notions do exist in Einsteinian theories, or in some particular solution of them, it is impossible to synthesize them coherently to construct a common entity which could be called *time*, as is the case in Newtonian physics.

In section 2, I analyze the collection of the main characteristics of Newtonian time, in relation with the notions mentioned above, whose synthesis precisely leads to the construction of time (through the property of datation). Then, section 3 introduces special relativity, and illustrates the disappearance of time there; first by showing that some of the above notions with temporal flavor have no counterpart; secondly by showing that the different remaining notions — causal structure, proper durations and time-functions — cannot be synthesized coherently to construct an entity with the properties of time. In section 3.1 I suggest an original historical analogy to illustrate this absorption of time into space-time. Section 4 introduces the general theory of relativity and, concentrating on the differences with the *special* theory, I show how the disappearance of time is still more crucial. I present (section 5) a specific analysis of time-functions in the theory that I illustrate (section 5.3) with the case of

[1] As it is well known, the Galilean or Newtonian physics is also *relativistic*, in the sense that it obeys the Relativity Principle. I will however conform to the usage of calling " special relativity " and " general relativity " the two Einsteinian theories. Hereafter, " Einsteinian" will refer to both theories where the status of the notions analyzed here are analogous. For instance, Einsteinian space-time refers both to the Minkowski spacetime of special relativity, and to the space-time of general relativity.

cosmology and cosmic time.

2 Newtonian time

“ What, then, is time? If no one asks me, I know what it is. If I wish to explain it to him who asks me, I do not know. Yet I say with confidence that I know that if nothing passed away, there would be no past time; and if nothing were still coming, there would be no future time; and if there were nothing at all, there would be no present time. ”
Augustine, (Chapter XV:17)

The success of Newtonian physics was largely due to the introduction of a frame for Physics: the Universe, unique and unified by definition. This is a prerequisite for the universality of the physical laws, which is itself a requirement for the scientific methodology; in other words, a necessary condition for physics: no physics without universal law (a prerequisite for experimental methodology) ; no universal law without an universal framework.

Newton defined this universal framework to be composed of space and time, whose properties are given in his *Principia.* In many aspects, the Newtonian conceptions about space and time agree with those of our intuition. This helps a lot for understanding them. On the other hand, this may be an obstacle for realizing that they do not represent the reality of the world; in particular that there is no analog to the Newtonian time in nature, as well expressed in the Einsteinian relativistic theories.

Since Newton, many observational and experimental results have shown to physicists that the real world is incompatible with the existence of Newtonian time. Although this is well known, I claim that it is incompatible with the notion of *time* itself, defined as an entity admitting a minimum collection of properties that we usually attribute to time. Although this is well taken into account by the Einsteinian theories of relativity, this is a property of nature which does not depend on any physical theory or model, and which is more and more confirmed by our progressing experimental and observational exploration of the world. How is it possible to reconcile this fact with our use of the notion of time in ordinary life ? Which time-related notions can be defined in relativistic theory? I will try to answer these questions.

Although nobody, in physics or in philosophy, has claimed to give a definitive and complete definition of time, we know quite well the properties of *Newtonian* time. Whatever we would like to call “ time ” should share at least a subsample of these properties. Thus, it is a prerequisite for the study of temporal notions in any theory to examine the properties of Newtonian time.

Many particular circumstances offer the possibility to define some entities with temporal flavor, i.e., which share such or such property of the Newtonian time (e.g., “ proper times”, or “ cosmic time”). I claim that we must resist the temptation of referring to them as time, since it is impossible to reconcile them with other properties that we also desire associate to time, without generating contradictory statements. This is what I interpret as a manifestation of the impossibility of existence of time.

2.0.1 The Newtonian space-time

A large part of this paper will compare some properties of Newtonian Physics with those of Einsteinian Physics. But it may be not easy to really appreciate the deep differences between the two theories, because they are masked by their different formulations. For that reason, it may appear convenient to express them in a common language, and to reformulate part of the Newtonian approach in order to make easier the comparison with Einsteinian physics. This does not change the physics, but only the way to express it, which may appear quite unfamiliar. Concretely, I will introduce a notion of a *Newtonian space-time*, to compare it with the Einsteinian space-time.

Both theories consider space-time as a geometric entity (thus a set with geometrical properties); the occurrence of a physical event is identified with an element (or point) of space-time. [2] This definition makes sense both in the Newtonian and in the Einsteinian framework. Their respective space-times admit different well defined geometrical structures, that I will not discuss here in details. The main property of Newtonian space-time (not shared by the Einsteinian one) is that it can be *uniquely* decomposed into a product of Space and a time-line. [3] This splits the Newtonian chrono-geometry into a spatial geometry plus a chronometry. The existence of Newtonian time is equivalent to a metric structure (chronometry) of the time line, entirely independent from the spatial geometry. The introduction of the Einsteinian theories results from the realization (first by Einstein) that the real world phenomena are incompatible with this conception.

2.1 A datation is a time-function

The first, and probably the more fundamental property of Newtonian time, is *datation*. As I will show, the whole collection of its other properties result from datation, and I will adopt the position that a minimal requirement for a notion of “ time” is to be – at least – a datation.

A datation is a process which assigns to any event a real number called its date. This is a function from space-time to the set $\mathbb{R}$ of real numbers, here called the time-line, what we equally call a *time-function*. The Newtonian datation is build-in the geometrical (fiber bundle) structure of Newtonian theory, where it appears precisely as a projection of space-time (with fiber bundle structure) onto its basis, the time-line (the possibility of such a projection disappears in Einsteinian theories).

[2] It is tempting to define space-time as the set of all events and some modern approaches adopt indeed this position. In our present vision of the world (Newtonian or Einsteinian), we admit the possibility of points of space-time (in Newtonian physics, a date and a location) where no physical event occurs; but since an event could occur (or have occurred) there, space-time may be seen as the set of *possible occurrences* of events.

[3] More correctly, it is defined as a *fiber bundle* manifold, with the time line as basis manifold.

The datation specifies immediately a unique and well defined total ordering of the events: the *chronological ordering* (more simply, the chronology) of space-time.

We will examine below the possible existence of time-functions in Einsteinian theories. A first important result is that some space-times wich are solutions of general relativity do not admit any time-function (some solutions, like the Gödel space-time[12], admit *local* but not *global* time-functions). As a second result, a space-time solution which admits a time-function does admit an infinite collection of them (even in the simplest case of Minkowski spacetime). One may wish to adopt a particular one (like for example the *cosmic time* in cosmology, see below), and to promote it to the status of " time ". We will see that this leads to three kinds of problems:
- this may be simply not be possible in some solutions of general relativity;
- when possible, such a choice is arbitrary. Different choices lead to different estimations of the date of an event, of the " duration " separating two events, and even (in some cases), of their chronological ordering; this means that, for two given events A and B (not causally connected, see below), one choice implies that " A is chronologically posterior to B " ; and the other that " B is posterior to A ".
- the chronological notions derived from a time-function usually contradict physical measurements: physical events with different dates may appear simultaneous (and vice versa); the " chronological duration " of a process, as derived from a time-function differs in general from its *proper duration*, obtained as the result of a physical measurement.

We will see in which particular circumstances it is possible, in the Einsteinian context, to chose a time function with a special status, and to which extent it may share some properties of the (Newtonian) time.

2.2 The properties of Newtonian time

The datation defines a *chronology* : a total ordering of all points of space-time, thus of all events, thanks to their dates. The chronological past (present / future) of an given event is well defined as the subset of events with a smaller (equal / greater) date.
The *causal structure* is defined as the relation which answers the question " which events are a possible cause of (can have an influence on) a given event ? ". The causal structure of Newtonian physics is strictly equivalent to its chronology, thus a total ordering.
Simultaneity : Events are (chronologically) simultaneous when they have the same date.
Space (at time t) is the subset of all events (or of points of space-time) which share the same date t.
Duration is a number which can be assigned to any physical process. It is a physical quantity, that we can measure with a clock.

2.2.1 Durations

I insist on that property since its status will be completely modified in Einsteinian theories. A duration is an assignment of a real number – its duration – to any process (that we also call a history).

In Newtonian physics, duration is defined from the datation : the duration of a process is the difference between the dates of its final and initial events. Hence, the important property:
(D) = the duration of a process only depends on its initial and final events.

The Einsteinian theories admit the notion of *proper duration* of a process: a unique real number, assigned to it by the metric of space-time. But it is not linked to any time function (or time) and does not obey the property (D) above: it depends not only on the terminal events of the process, but on the details of its whole history as illustrated by the famous twin " paradox ". [4] In particular, when a *time function* is defined, the proper duration of a physical process is *not* the differences between its values (the " dates ") at the terminal events.

3 Special relativity

The best characterization of the relativistic revolution is the replacement of space + time (which together form what we have called Newtonian space-time) by a 4-dimensional space-time as the frame for physics. Although Newtonian space and time each admit a separate metric, Einsteinian space-time admits an unique spatio-temporal metric: this is a 4-dimensional Lorentzian manifold, i.e., a differentiable manifold with a Lorentzian metric (see below). Any notion with temporal flavor is defined from the space-time metric, not from a temporal metric.

The space-time of special relativity is Minkowski spacetime, the only 4-dimensional flat Lorentzian manifold. [5] It is homogeneous and isotropic, which means that it admits a maximal group of isometries. [6]. Its isotropy implies the complete equivalence of all time-like directions. In a Riemanian manifold (like Newtonian space), isotropy means the equivalence of all possible directions, in the sense that they can be exchanged by a rotation without modifying any property. In a Lorentzian manifold, this is slightly different since the metric divides the possible directions into three families : space-like, time-like and light-like, see below. Isotropy implies the equivalence of all time-like directions. Newtonian space-time is *not* isotropic since it admits a unique time-line which has a special status compared to all other directions

The choice of a time function (i.e., of a datation) selects an arbitrary direction in space-time and breaks this isotropy (conversely, the selection of a temporal direction defines an infinite family of datations). The infinite number of different possibilities to accomplish such a breaking leads to an infinite number of possible time functions, leading to mutual contradictory statements about temporal notions (see below).

[4] originally proposed by Paul Langevin. It appears, however, as a paradox only when one tries to reconcile it with a notion of time.

[5] up to topological variants [18].

[6] In a [pseudo-]Riemanian manifold, an isometry is a transformation (diffeomorphism) which preserves the metric. Any [pseudo-]Riemanian manifold admits a group of isometries. When it is maximal, the manifold is said to be homogeneous and isotropic

3.1 A Newtonian analogy

The isotropy of Minkowski space-time is a fundamental fact of relativistic physics, and the deep expression of the non existence of time. For this reason, I will present an analogy[7] between *the status of time in Minkowski spacetime* and *the status of the vertical in Newtonian space*: an analogy between *the transition from Aristotelician Physics to Newtonian Physics* and *the transition from Newtonian Physics to Einsteinian Physics*.

3.1.1 From Aristotle to Newton

- Pre-Newtonian physics distinguishes horizontal and vertical directions. Horizontal directions form a 2-dimensional isotropic plane; the-1 dimensional vertical direction is exterior to it.
  This is for instance manifest in the Aristotelian claim that the natural motion (in the sublunar world) is vertical. This may be called a fundamental *anisotropy* of pre-Newtonian space. [8]
- The Newtonian transition is the replacement
  horizontal (plane) + vertical $\rightarrow$ *3-dimensional isotropic space*.
  This isotropy means that all dimensions have the same status. This goes against the empirical evidence since our main experience is terrestrial: we have no chance to experience the isotropy of Newtonian space since it is masked by the gravitational field of the Earth. One aspect of the Newtonian revolution was precisely to disentangle gravitational effects from spatial geometry.
- All rotations are permitted in Newtonian physics, including those mixing vertical and horizontal dimensions.
- There is no way (except in the local terrestrial environment) to select a vertical among all the spatial directions. The vertical appears as the signature of something new, and exterior to geometry of space: the local gravitational field.
- There is no meaning of the vertical in empty space: it can be any direction and there is no canonical way to select one.
- In a group theoretical formulation, the rotation group SO(2) of 2-dimensional plane is improved to the rotation group SO(3) of 3-dimensional space. The choice of a particular direction (e.g., to be called the vertical) is a breaking of the corresponding SO(3) symmetry; or, equivalently, a group reduction SO(3) $\rightarrow$ SO(2).

3.1.2 From Newton to Einstein

Replacing " horizontal / vertical " by " space / time "; and " space " by " space-time "; everything can be imported to depict the Einsteinian revolution.

- Pre-Einsteinian Physics distinguishes space and time. Space forms a 3-dimensional isotropic manifold; the one-dimensional time-line is

[7] that I have also developed in [14].
[8] although, strictly speaking, the notion of *space* is an anachronism in this context.

exterior to it. This may be called the fundamental *anisotropy* of pre-Einsteinian space-time.

- The Einsteinian transition (special relativity) :
  space + time $\rightarrow$ *4-dimensional isotropic space-time*.
  This isotropy means that all time-like dimensions have the same status.

  Again, this goes against the empirical evidence since our main experience distinguishes space and time. We have no chance to experience the isotropy of space-time, since it is masked by our impossibility to reach high velocities (compared to c). Einsteinian theories account of this as a consequence of our particular status, being unable to suffer high acceleration and to reach high velocities.
- All rotations are permitted in Einsteinian physics, including those mixing spatial and temporal dimensions: they are called *boosts* (or inertial transformations).
- There is no way (except in special conditions to be examined) to select a " time " among all the (time-like) directions.
- In empty space-time, any time-like direction can be chosen as a substitute of time; there is no way to select one. One may wish to interpret this as " Time is nowhere ", or as " Time is any direction in space-time ", see below.
- In a group theory formulation, the rotation group SO(3) of 3-dimensional space is improved to the rotation group SO(1,3) of 4-dimensional Minkowski space-time, the Lorentz group.

  The choice of a particular direction (implied by that of a time function) is a breaking of the corresponding Lorentz symmetry; or, equivalently, a group reduction SO(3,1) $\rightarrow$ SO(3), from the Lorentz group to the spatial rotation group.

3.1.3 Selection: vertical and time functions

In empty space, all directions are equivalent. How to select one? Where is the vertical ? Maybe I would like to refer to the direction to my (distant) planet Earth ? Or to the nearest planet, or star, of which I am feeling the gravitational attraction ? Or, more simply, the direction to my space ship ? Or that of my feet ? ...

In any case, a choice involves something exterior to the geometry; something *in addition* to space, usually linked to a gravitational force.

The situation is exactly similar for the direction of time in space-time: there are infinitely many equivalent possible directions which may be chosen as time-functions. They lead to contradictory notions of, e.g., chronology. In any case, something exterior to the chrono-geometry is needed to select a direction: like the presence of some material system. The problem is that different matter components define different and incompatible " time-functions ". The choice, in Einsteinian space-time, of a time direction which coincides with my proper time along my own world-line (see below) has the same status than the choice, in Newtonian space, of the vertical as the direction to my feet.

4 Einsteinian Relativistic theories

An Einsteinian theory, and more generally a metric theory, considers that the frame of physics is a space-time: : a 4-dimensional Lorentzian manifold, i.e., a differentiable manifold with a Lorentzian metric (and some additional properties). To this metric – which is a tensor – is associated a curvature (also a tensor) which expresses the“ shape ” of space-time.

- In *special relativity*, the metric is entirely specified and has no curvature (it is flat). It is called the Minkowski metric, and the space-time is called the Minkowski space-time. Gravitation is not taken into account.

- In *general relativity*, the Lorentzian metric has a curvature identified with the gravitational field. It is not fixed a priori but is obtained “ dynamically ” as a result of the field equations: the Einstein equations. A basic problem is to find the metric of space-time, for a given configuration of matter-energy, by solving the Einstein equations (or another equation in some other metric theory).

4.0.4 The metric

By definition, a metric is a tool which assigns a quadratic-interval (QI) to any curve segment (rigorously, to any vector). We are used to Euclidean (or Riemanian) metrics, where the always positive QI leads to the usual definition of length (as its square root). But the relativistic theories use a *Lorentzian* metric, which assigns a QI of any sign.

This leads first, in both Einsteinian theories, to a classification of the curves[9]: they are called time-like, light-like or space-like when their QI is positive, zero, or negative (the opposite convention also exists). This is at the origin of the causal structure, see below.

This leads also to the definition of the *proper duration* of a segment of a time-like curve: this is the square root of its QI. [10] Proper duration is an attribute of a curve segment provided by the space-time metric. It is not linked to any time, datation or time-function.

4.1 Time related notions in GR

Time and space are absent in the vocabulary of relativistic theories; as well as velocities. [11] The metric of space-time leads however to the definition of various quantities and structures with a temporal connotation.

- A causal structure (see next section) is a well defined causal ordering in space-time, imprinted by the metric.
- A *proper duration* is assigned to each segment of a time-like curve; in other words, to any history.

[9] A curve is of a given type when its tangent vector always remains of that type.

[10] Similarly, the *proper length* of a space-like curve segment is the square root of minus its QI.

[11] *Four-velocities* are however well defined in space-time, and usually simply called “ velocities ”.

- The time-related notion of *redshift* is well defined and very useful in relativistic theories.
- A space-time may or may not admit time functions, defined in section 5. When it does, it admits an infinite number of them. Each may share *some* properties of Newtonian time: it defines a datation, an associated chronology, an associated simultaneity... But these notions can be defined in an infinite number of different manners, which contradict each other.

Contrary to the case of Newtonian physics, these notions are not mutually compatible. For instance,

- the chronology defined from one datation differs from the chronology defined by another; and it does not coincide with the causal ordering.
- In two space-times admitting *the same* causal structure, the proper durations assigned to the same history are (in general) different.
- Given a time function, the proper duration of a history is not the difference between the terminal and initial dates of the history.

4.2 The causal structure

Any (relativistic) space-time admits a well defined *causal structure*. It is defined from its Lorentzian metric [12] through the following steps:

- The Lorentzian metric assigns (by definition) a quadratic interval (QI) to any curve segment (rigorously, to any vector).
- This classifies the segments as time-like, light-like or space-like according to the sign of their QI (positive, zero or negative respectively). [13]
- By extension, this also classifies the curves; only the curves all of whose segments (all tangent vectors) keep the same sign are retained as physically significant.
- Non space-like curves are called *causal*.

This establishes the possible causal relations between two events A and B (points in space-time):

- A causally precedes B ($A \leq B$) if there is a future-directed [14] causal curve from A to B (and reciprocally);
- A and B are causally disconnected, if there is no causal curve joining them.

[12] in fact from its *conformal part* only. Two different metrics are said to be *conformally related* where one is equal to the other multiplied by a scalar function; a conformal structure is an equivalence class under this relation.

[13] A conformal structure does not associate a QI to a segment, but only a sign. This is sufficient to establish the present classification.

[14] space-time is assumed to be oriented, and time-oriented.

This order relation is the causal structure of space-time. A main difference with the Newtonian case — where the third possibility is absent — is that the ordering is *partial* rather than total. An immediate consequence is the impossibility to associate to an event something which can be called its *present*.

The causal structure is well defined in any space-time. It is completely independent from any notion of temporal character, like time-function, datation, chronology or proper durations...

It may be important to remark that the presence of a well defined causal structure – which is always guaranteed in any space-time – does not forbid what are called " chronology violations " (also sometimes " causality violations "); like, e.g., time travel. In fact, *time travel* is possible in a given space-time, only if no (global) time-function exists. As a consequence, time travel is incompatible with the existence of time. The purpose of Gödel, when he exhibited his solutions of general relativity with the possibility of time travel, was precisely to illustrate the impossibility of the existence of time [7, 12, 30, 11, 2].

In general relativity, the causal structure is defined from the metric. Some present speculative approaches try to define space-time without a metric, but retain the causal structure only. Then, it is intrinsically defined as an order relation in the set of events, without any reference to a metric. [15] This is for instance at the basis of the *causet* (for " causal set") approaches (see, e.g., [6], [10], [27], [28]).

4.3 World lines and Proper time

4.3.1 The world line

The fundamental rule of relativistic theories is that
a material object (in particular an observer) follows a time-like line in space-time, called its world line.
This is the line of its successive positions in space-time. Reciprocally, each time-like line in space-time may be seen as the world line of a potential observer. More condensed, in a chrono-geometrical language, a physical system (a particle, an observer ...) *is* its own world line.

A physical process is a continuous succession of events experienced by a physical system like an observer [16]: a part of his life, represented by a portion of his world line, represents a history. The metric of space-time assigns to any history — a portion of a time-like curve — its *proper duration*.

The proper duration is the real physical quantity that an observer (or physical system) may experience and call a duration in the usual sense. A physical measurement, using any kind of clock, always gives the proper duration of the clock history (which is also mine if I am close to the clock). My physiological time (the beat of my heart); my intellectual time (e.g.,

[15] thus, without the possibility to consider proper durations, proper times, or time functions.

[16] I call *observer* any physical system which is able to record something, in particular its proper time; an observer is typically a human being; but this can be any device with recording ability. Since an atom itself may have transitions resulting from an interaction with,e.g., radiation, it is sometimes considered as an observer.

the one necessary for a mental calculation) etc. are linked to my proper time. This is a fundamental assumption of the relativistic theories. Note again that this has no relation with any kind of time-function.

A important point is that an observer may experience — or measure — proper durations along his own world line only (where he stands); the notion has absolutely no meaning elsewhere in space-time. There is no way, and this has no meaning, to try to compare the proper times of different observers, except in the trivial case when they share (approximately) the same world line.

A fundamental difference with Newtonian physics comes from the fact that the proper duration of a history *is not fixed by its initial and final events*: it depends on the whole corresponding trajectory (world line) in space-time; and does not obey the property (D) mentioned above. Between two given events, there is however one unique [17] curve segment (a history) which has the *longest* proper duration : the geodesic. [18] This is a straight line when space-time is flat (e.g., Minkowski spacetime). In the case where a datation has been defined, the proper duration is *not* the difference of dates between its terminal events. The *Langevin twin " paradox "* is an illustration of this situation: between the same initial and final events, the two twins have experienced different (proper) durations: their ages are different when they meet again. More generally, between two specified events, two observers $\mathcal{A}$ and $\mathcal{B}$ experience different proper durations associated to their different histories.

In the general case, the *redshift* is a tool allowing to compare the different *perceptions* of a given process by two different observers. But it would have no meaning to say that one proper time is flowing faster or slower than an other; or that one duration is contracted or dilated w.r.t. the other.

4.3.2 Proper time

A given observer $\mathcal{O}$ may define a proper time flowing along his world line only (that I also write $\mathcal{O}$). After the choice of an arbitrary origin O (a point on the world line), its value for an event A of $\mathcal{O}$ is defined as the proper duration separating A from O, counted with a sign which depends on the time-orientation. This proper time is defined on $\mathcal{O}$ only, not in the universe outside. Thus it does not define a datation, and provides no chronology, for events out of $\mathcal{O}$.

A selected observer (like " me ") may wish to extend the validity of his proper time outside the whole universe. He will search for a time-function which, when restricted to his world line $\mathcal{O}$, coincides with his proper time. There is an infinite number of different manners to perform such an extension: this requirement selects an infinite sub-family among the infinite family of possible time functions. Two observers select different sub-families (in some particular cases — like inertial observer in

[17] Except in specific situations like gravitational lensing or a multi-connected universe [18].

[18] This is in exact analogy with the case of Riemanian geometry in space: between two points of *space*, there is a line with *shortest* length: the geodesic; a straight line in flat Euclidean space.

Minkowski spacetime — there is a canonical way to privilegiate one, see below).

5 Time functions in relativistic theories

The Newtonian time is a datation, i.e., a time-function. Proper times or durations do not define datations. Do the relativistic theories admit time-functions ?

The definition remains the same : a function which assigns to each event a number, and which increases along each future directed time-like line. [19] This means that such a time-function “ flows ” for each objet along its world line.

A time function generates a foliation of space-time: a decomposition of the type of a product Space $\times$ Time, the latter being identified to the time function. [20] The “ spatial sections ” are the level hypersurfaces of the time-function. Thus, a first condition for the existence of a time-function is that the space-time admits such a splitting. This is not always the case, and an important result is that some space-times – which are solutions of general relativity – do not admit any time function.
When a time function does exist, there is always an infinity of them. [21] We examine below when, and how, is it possible to make a pertinent selection among this diversity. This is analog to the search for a vertical direction in the isotropic Newtonian space and, similarly, this requires (in general) a reference external to the chrono-geometry of space-time.

We insist on the fact that a time-function does not coincide, in general, with the proper time of an observer along his world line; and that the proper duration of a process (what is measured by its clock) differs from the corresponding lapse of a time-function.

5.1 Time functions in special relativity

The Minkowski spacetime admits an infinity of possible decompositions under the form of a product time $\times$ space. Each generates an infinity of time-functions, related by reparametrizations.

Since Minkowski spacetime is flat (without curvature), it seems reasonable to require that the spatial sections are flat also (i.e., each identical (isometric) to the Euclidean space $\mathbb{R}^3$). This is equivalent to require that the time-lines are straight lines. This constraint reduces the plurality of time-functions, but still leaves an infinity of them, which lead to different and contradictory datations, chronology, simultaneity.

A further step may be to require that the time function coincides with the proper time of an inertial observer, along his world line. This

[19] That the time-function increases along the world line of an observer does not imply that it is identical to his proper time: it can be any increasing function of the proper time.

[20] A time function generates an unique foliation; a foliation generates a family of time functions related by reparametrizations.

[21] Analogy: in the Newtonian isotropic space, there exits an infinity of equivalent directions which may be called the vertical; here, the infinity of arbitrary possibilities is still much “ larger ”.

reduces again the choice; but still leaves an infinity of possibilities, corresponding to all possible spatial orientations of an observer. The isotropy of Minkowski spacetime guaranties – by definition – that all these directions are equivalent: all these time-functions share exactly the same status; none of them plays a privileged role.

A final selection requires the choice of a particular inertial observer. A natural choice is " me ", assuming that I am inertial, which is a reasonable approximation when high precision is not required. Thus I may chose *the* time function coinciding with *my* proper time along my world line (and defining flat spatial sections). This corresponds, in the context of special relativity, to the choice made by Newton, that he called *universal time.* (a more correct appellation would be " *my* universal time ".) But if I am observing a galaxy, for instance, it could be more convenient to chose the " universal time " of that galaxy. This would define a completely different chronology, not compatible with the first one.

In any case,
- a different observer, having naturally chosen the time function coinciding with *his* proper time along his world line, would adopt a chronology entirely different from mine: he would assign different dates to the events, and even different chronological ordering. His notion of simultaneity would contradict mine.
- the proper duration of a process (like the life time of a star, or of a particle...) never coincides with the lapse of (my) universal time between birth and death. Except for the history of my own life, since I defined the (my) universal time for that purpose.

5.2 Time functions in general relativity

The Minkowski spacetime is only a very crude approximation to the real world, since it does not take gravitation into account. In general relativity, presently the best theory to describe the world, the situation is less simple.

First, the theory admits solutions where no time function may exist (globally [22]). A case is given for instance by the Gödel's solution, that he constructed with the specific purpose of showing explicitly the impossibility to define time in general relativity [12, 30]. There are many other solutions of the theory in which no time function may exist. Most of them allow the possibility of time travel (whose physical pertinence is still under discussion today) (see, e.g., [11, 2, 15]).

The majority of the solutions of general relativity which have been proposed to describe realistic situations, admit however *an infinity* of global time-functions. Since they lead to contradictory datations, chronologies, and notion of simultaneity, this raises again the question of selecting one. The situation is similar to that in special relativity presented above, although with some differences.

[22] For most of these solutions, it is however possible to define *local* time functions; i.e., valid in some limited region of space-time. When restricted to such a limited region of the universe, the discussion is similar to that below

5.2.1 Spatial sections with constant curvature

In special relativity, we applied a first prescription by selecting those time functions leading to spatial sections with zero curvature. This is not possible (in general) in the curved space-time of general relativity. A less severe prescription would consist in requiring spatial sections (level hypersurfaces) to verify some symmetry property, namely being of constant curvature (homogeneous and isotropic). Only a very limited class of solutions of the theory allows this possibility: the corresponding space-times are said to obey the *cosmological principle* (CP); this is the definition. The family of such solutions constitute the *Friedmann-Lemaître models*.

For a space-time obeying the CP, we may require in addition that the time-function coincides with the proper time of an inertial observer. This still reduces the variety of time-functions. But I will not go deeper into the details of the procedure because of the following remarks.

First, this applies only to the very limited class of space-times obeying the CP. This is not the case of our *real* universe, where galaxies and galaxy clusters imprint local curvature at various places of space-time, so that no spatial sections with constant curvature can be found: the cosmological description of our Universe with the popular Friedmann-Lemaître models (including the big bang models) is a zero order approximation only, which neglects all the curvature fluctuations imprinted by the cosmic structures. The validity of a time-function selected through the constant curvature requirement cannot be extended beyond this zero order approximation. For realistic cosmology, its limited pertinence is very difficult to evaluate.

Quite fortunately, another type of selection procedure may be applied to (most of) these models: that of the *cosmic time* (defined below). At the zero order (i.e., for the unperturbed cosmological models), it coincides with the constant curvature procedure above; but it extends nicely to the (perturbed) real situation. Thus, the constant curvature procedure appears useless [5].

5.3 Cosmic time

Given a space-time, the *cosmic time* is a particular time-function defined in the following way: its value at an event X is the supremum of all the proper durations of all future directed time-like curves ending in X.

Not every space-time admits a cosmic time. For instance, Minkowski spacetime does not, since such a supremum has an infinite value for any event. When it is defined (taking finite values), it is by construction strictly growing along each future-directed time-like curve.

Cosmic time is well defined for the expanding Friedmann-Lemaître models at the basis of our cosmology, at least for those with a big bang (see, e.g., [5]), through their metric. For a specific class of (imaginary) inertial observers, called *comoving*, it coincides for each with his proper-time along his world line. The terrestrial observer is, in first approximation, one of these comoving inertial observers. The cosmic time, like any time function, defines a congruence of time lines everywhere tangent to the gradient of the time function: the world *congruence*. One may imagine that space-time is filled of (imaginary) comoving observers following these lines.

5.3.1 Material cosmic time

This suggests an alternative option [26] to define a " material cosmic time" as the function which identifies with the proper time of each (imaginary) observer of this congruence, along his worldline. This is a particular case of the general possibility to define a time function from a material content. Strictly speaking, this requires however that the corresponding matter (conveniently called a *cosmic clock*) occupies *the totallity* of the points of space-time, in order that the world lines form a *congruence* and that the time-function is defined everywhere. We may wish, for instance, to consider the collection of galaxies present in our Universe as such a cosmic clock. But the time-function would remain undefined at the points where no galaxy is present and it is very difficult to define an averaging process in curved space-time (see more in [26]) to restore the lacking information. In addition, galaxies are subject to proper motions (which superpose to the cosmic expansion), so that their proper times, flowing along their world lines, would not coincide with the cosmic time according to the first definition: for each galaxy, there would be a local redshift factor between the two " cosmic times " corresponding to the two definitions, to which no observation could give access.

But the main objection comes from the impossibility to have any observational access to material cosmic time according to its definition. Even if we assume that galaxies (or other objects) fill the totality of space-time, and if we do not worry about their proper motions, there is no possibility to measure the proper time along a given world line for an observer outside that world line. This makes this definition useless in practice. However, the formal *existence* of such " material time functions ", even if they remain out of measurement possibilities, is an important fact which plays a role for handling the " problem of time " in quantum gravity (see below).

5.3.2 Validity of cosmic time

Coming back to the original definition, cosmic time, like any time-function, defines spatial sections (its level hypersurfaces which foliate space-time). [23] For the Friedmann-Lemaître models – which obey the CP – these sections have constant curvature, so that the cosmic time precisely obeys the constant curvature criterion mentioned above. Its advantage lies in the fact that its definition still holds in a *perturbed* cosmological model, which describe our real universe.

The cosmic time applies only to a restricted set of solutions of general relativity, which seem however well adapted to describe our whole universe. Cosmology mostly uses in fact an " averaged cosmic time " : defined from a strict Friedmann-Lemaître model which is only a zero order approximation, an averaged version, of our real universe. An exact – but unknown – version of cosmic time does exist, which would provide a convenient chronology for our real, perturbed, Universe.

[23] The world lines of comoving matter are orthogonal to these sections.

This assumes however that our Universe is well described by a so called big bang model with an initial singularity. Most cosmologists however estimate today that this is not the case, and that the present phase of cosmic expansion did not begin in such a singularity but may result, for instance, from a *cosmic bounce* following a phase of cosmic contraction. The corresponding space-time would admit no cosmic time, although some other cosmic time functions (see below) may remain defined, at least in some part of space-time.

5.3.3 Cosmic time is not a time

Assuming that cosmic time is well defined, it provides a useful cosmic chronology, which is its main advantage. For instance, the " age of the universe " t_U is its value here and now (thus, the extremum of all the proper durations of all future-directed time-like curve ending here and now). This is perfectly well defined, and implies that no object can have an age greater than t_U. It should be realized however that
- the proper duration of a cosmic process (say, the life of a star) as physically measured by a clock *is not* (in general) the difference between the cosmic time values of end and beginning. It may differ by a very important factor (which tends to infinity for a very quickly moving object) .
- For a non comoving observer, the cosmic time differs with his (physical) proper time — the only that he has the capacity to measure — along his world line (also by an important factor).
- Different events sharing the same value of the cosmic time *are not* simultaneous according to the Einstein's synchronization procedure, even performed by comoving observers [16] (this is still worse for non comoving observers).

The cosmic time has a local physical pertinence for the terrestrial observer since it coincides with his " universal time " along his world line. But this universal time is well defined from proper durations and the reference to cosmic time is unnecessary in that purpose. Moreover, cosmic time coincides with universal time only in the approximation that the terrestrial observer is comoving.

The main defect of cosmic time, however, is the impossibility to have access to it directly: if we consider a cosmic event, like the explosion of a supernova in a remote galaxy, no physical measurement provides its value. The latter may only be obtained through an indirect reconstitution from the redshift measurement. The latter suffers however two important drawbacks. First, un unknown component of the redshift is not cosmological and caused by the proper motion of the observed source. This introduces an error and the best we can do (and that is done in fact) is to assume that such errors average to zero for a statistical population of galaxies. This requires however to consider averaging processes which are known to suffer from biases. At best, cosmic time estimations could be considered to be " true on average ". The second drawback results from the fact that the conversion from a redshift (even assumed perfectly " cosmological ") to a value of cosmic time requires a perfect knowledge of the cosmological model, i.e., of the shape of space-time. Given our uncertain knowledge of the Hubble constant, of the deceleration parameter and of the infinite list of similar cosmological parameters, we must realize that an assigned value of cosmic time to an observed event is model dependent.

5.3.4 Other cosmic time functions

The cosmic time being not measurable, only (at best) statistically defined and model dependent, it would be wise to consider it as an useful convention to describe the chronology of the cosmic events rather than as physically relevant quantity, in any case certainly not a *time*. For this reason, cosmological calculations most often use other time-functions.
- The *conformal time* has properties comparable to those of the cosmic time. It does not identify with the proper times of inertial observers but it has more direct links with the causal structure of space-time than the cosmic time and this is the main reason of its preferred use in cosmological calculations. It also has the nice property (not shared by cosmic time) that different events sharing the same value of the conformal time are seen as simultaneous by a comoving observer applying the Einstein's prescription [16]. It is however – like the cosmic time – not directly measurable.
- The (properly normalized) *scale factor* of the universe may be seen as a convenient time function. It offers the same advantages as the cosmic time, except that it does not coincides with the proper times of inertial observers. Its value τ is however directly accessible from observations. Namely, for a source (galaxy) observed with redshift z, $\tau = \frac{k}{1+z}$, where k is a normalization constant which may be is conveniently chosen as the present " age of the Universe " t_U. One can also conveniently use the logarithm of that quantity. It is also linked to the Cosmic Microwave Background temperature T through the relation $\tau = t_U \ \frac{T_0}{T}$, where $T_0 \approx 2.7K$ is the present temperature of the cosmic radiation. It also suffers from the fact that the measured redshift is not cosmological but has an unknown proper component.

Thus, even in the simplest situation of a space-time obeying the CP, different time functions may be chosen. Each choice assigns " dates " to the cosmic events, but they contradict each other. [24] What is the preferred choice is a pure question of taste (*cosmic time* seems good for popularization) but the best attitude is probably no choice at all since it is perfectly possible to perform any cosmological calculation or reasoning without reference to any time function. Any choice (cosmic time or other) would remain a convention, more or less adapted to such or such study, and cannot pretend to have the status of a physical quantity and to offer a notion of time. A confusion of such notion with time would be a possible cause of mistake: a neutron, whose (average) life time is of the order of 14 minutes, may perfectly experience a lapse of several years of cosmic time; a star living one million year may perfectly subsist during a lapse of several millions years of cosmic time *etc* .

[24] For instance, in any big bang cosmological model, the recombination takes place at a fixed value of redhift-time, namely t_U/z_{rec}, where $z_{rec} \approx 1\ 100$. In a model without inflation, the value of the conformal time at recombination is of the same order of magnitude; in a model with inflation, it is almost infinite. This discrepancy is a direct expression of the effect of inflation (see, e.g., [4]).

6 Time and general relativity

To summarize, general relativity admits two well defined notions which are usually associated to time: the causal structure and the proper durations. This is not sufficient to define a notion similar to time, which requires at least the existence of a time-function.

First, the existence of solutions of general relativity admitting no time-functions implies that the notion of time is not generically defined in the theory and does not belong to its ontology. This has been emphasized by many authors, in particular by Gödel [7].

On the other hand the theory admits many solutions sufficiently regular to allow time-functions. Selecting one requires however the adoption of a particular and subjective point of view, linked in general to a specific observer, with additional assumptions like for instance that of a specific matter component filling the universe. Even at this price it remains that, even in the most favorable situations, chronological propositions deduced from a time function cannot be reconciled with physical measurements of a temporal nature, in particular proper durations.

This impossibility is deeply rooted in the Einsteinian theory and is one of its most fundamental properties: the clearest manifestation of the impossibility to include the notion of time in its ontology: any philosophical attempt to interpret the reality of the world in conformity with our present physics must renounce the notion of time.

In a specific situation, it is always possible to chose a time function or another; for instance compatible with the proper time of an observer, or of an observed system, or obeying an alternative convenient prescription. Some authors like this old-fashioned way but it is dangerous since a confusion with *time* would be a source of errors.

For precise astronomical positioning, spatial navigation or communication (like for the decoding of the *Global Positioning System* (GPS) signals), there is no adapted choice of a time function. In such situations, one may use convenient space-time coordinates without any spatial or temporal character like for instance *radar* or *GPS* coordinates [8, 17, 19]. The study of the possibilities of time travel (involving explicit solutions like the Gödel Universe [12], with strongly counter-intuitive implications) is also an active field in general relativity. It is completely incompatible with any time function (since the possibility of time travel excludes the possibility of their existence [15]) and the relevant tool is the causal structure of space-time.

Let me finish by shortly mentioning some aspects of present research in fundamental physics with the quest for a new theory with more unifying power; which could, e.g., reconcile quantum and (general) relativistic physics. Such an enquiry has to face very seriously the non existence of time. This is the case for the search of a theory of *quantum gravity* [29], where the so called " problem of time " [13, 1, 20] has (at least) two facets. First, our usual view of quantization requires a notion of time, which is absent in the relativistic context. [25] This is one of the most serious dif-

[25] An aspect of this non existence is technically expressed by the time-reparametrization invariance, a facet of the covariance of the general relativity theory.

ficulties for the tentative quantization of gravitation. [21][26] Secondly, a quantum space-time (if such a thing does exist; it is precisely the goal of quantum gravity to define it) would admit no defined metric (but only a “ fluctuating one ” in some popularized language). Thus, even the notions of causal structure and proper durations (which, in general relativity, are imprinted by the metric) disappear: not only time is absent in quantum gravity (if such a theory does exists), but also the notions with temporal flavor that we have encountered in general relativity.

This motivates a collection of speculative answers for filling this gap. Let us mention the *relational time* [23, 3], or clock time (or semi-classical time) [27], which may to some extent play the role of a time function. The *thermal time* (a case of *emergent time*) [25, 9, 24] may provide a way to construct some analogous to proper time in the absence of a well defined classical (non quantum) space-time... These are attempts to define a physical notion with some validity in the context of a future theory, from which time-related notions could emerge.

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